LESSON 3.1

Solving Quadratic Equations by Taking Square Roots

Common Core Math Standards

The student is expected to:

COMMON CORE **N-CN.A.1**

Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. Also A-REI.B.4b

Mathematical Practices

COMMON CORE **MP.4 Modeling**

Language Objective

Have students decide whether a given square root is an imaginary number (square root of a negative number) or a real number and explain their reasoning to a partner.

ENGAGE

Essential Question: What is an imaginary number, and how is it useful in solving quadratic equations?

Possible answer: An imaginary number has the form bi; b is a nonzero real number and i is the imaginary unit, which is defined to be equal to $\sqrt{-1}$. Imaginary numbers allow you to solve quadratic equations of the form $x^2 = a$ when a is a negative number.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss the photo and how a quadratic function can be used to model a suspension bridge. Then preview the Lesson Performance Task.

Name ______ Class ______ Date ______

3.1 Solving Quadratic Equations by Taking Square Roots

Resource Locker

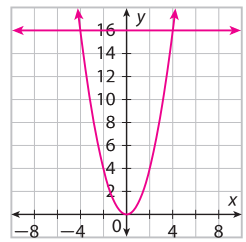
Essential Question: What is an imaginary number, and how is it useful in solving quadratic equations?

Explore Investigating Ways of Solving Simple Quadratic Equations

There are many ways to solve a quadratic equation. Here, you will use three methods to solve the equation $x^2 = 16$: by graphing, by factoring, and by taking square roots.

(A) Solve $x^2 = 16$ by graphing.

First treat each side of the equation as a function, and graph the two functions, which in this case are $f(x) = x^2$ and $g(x) = 16$, on the same coordinate plane.



Then identify the x-coordinates of the points where the two graphs intersect.

$x = \boxed{-4}$ or $x = \boxed{4}$

(B) Solve $x^2 = 16$ by factoring.

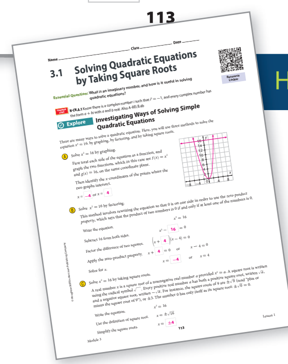
This method involves rewriting the equation so that 0 is on one side in order to use the *zero-product property*, which says that the product of two numbers is 0 if and only if at least one of the numbers is 0.

Write the equation.	$x^2 = 16$
Subtract 16 from both sides.	$x^2 - \boxed{16} = 0$
Factor the difference of two squares.	$(x + \boxed{4})(x - 4) = 0$
Apply the zero-product property.	$x + \boxed{4} = 0$ or $x - 4 = 0$
Solve for x.	$x = \boxed{-4}$ or $x = 4$

(C) Solve $x^2 = 16$ by taking square roots.

A real number x is a *square root* of a nonnegative real number a provided $x^2 = a$. A square root is written using the radical symbol $\sqrt{}$. Every positive real number a has both a positive square root, written $\sqrt{a}$, and a negative square root, written $-\sqrt{a}$. For instance, the square roots of 9 are $\pm\sqrt{9}$ (read "plus or minus the square root of 9"), or ± 3. The number 0 has only itself as its square root: $\pm\sqrt{0} = 0$.

Write the equation.	$x^2 = 16$
Use the definition of square root.	$x = \pm\sqrt{16}$
Simplify the square roots.	$x = \boxed{\pm 4}$



HARDCOVER PAGES 83–90

Turn to these pages to find this lesson in the hardcover student edition.

Reflect

1. Which of the three methods would you use to solve $x^2 = 5$? Explain, and then use the method to find the solutions.
   The graphing method would give only approximate solutions, while the factoring method can't be used because $x^2 - 5$ isn't a difference of two squares. However, taking square roots gives $x = \pm\sqrt{5}$.

2. Can the equation $x^2 = -9$ be solved by any of the three methods? Explain.
   None of the three methods can be used. Attempting to use the graphing method results in a parabola and a line that don't intersect. Attempting to use the factoring method results in the expression $x^2 + 9$, which isn't factorable. Attempting to use square roots doesn't make sense because square roots of negative numbers aren't defined.

Explain 1 Finding Real Solutions of Simple Quadratic Equations

When solving a quadratic equation of the form $ax^2 + c = 0$ by taking square roots, you may need to use the following properties of square roots to simplify the solutions. (In a later lesson, these properties are stated in a more general form and then proved.)

Property Name	Words	Symbols	Numbers
Product property of square roots	The square root of a product equals the product of the square roots of the factors.	$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ where $a \geq 0$ and $b \geq 0$	$\sqrt{12} = \sqrt{4 \cdot 3}$ $= \sqrt{4} \cdot \sqrt{3}$ $= 2\sqrt{3}$
Quotient property of square roots	The square root of a fraction equals the quotient of the square roots of the numerator and the denominator.	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ where $a \geq 0$ and $b > 0$	$\sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{\sqrt{9}}$ $= \frac{\sqrt{5}}{3}$

Using the quotient property of square roots may require an additional step of *rationalizing the denominator* if the denominator is not a rational number. For instance, the quotient property allows you to write $\sqrt{\frac{2}{7}}$ as $\frac{\sqrt{2}}{\sqrt{7}}$, but $\sqrt{7}$ is not a rational number. To rationalize the denominator, multiply $\frac{\sqrt{2}}{\sqrt{7}}$ by $\frac{\sqrt{7}}{\sqrt{7}}$ (a form of 1) and get this result: $\frac{\sqrt{2}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{14}}{\sqrt{49}} = \frac{\sqrt{14}}{7}$.

EXPLORE

Investigating Ways of Solving Simple Quadratic Equations

INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

MP.3 Discuss whether 1 can be used instead of 0 in the Zero Product Property to make a "One" Product Property. Have students show that if $ab = 1$, neither a nor b must equal 1. Have them consider −1 and other numbers and lead them to conclude that the property holds only for 0.

QUESTIONING STRATEGIES

How can you use the factored form of a quadratic equation to find the zeros of the related quadratic function? **Factor the equation, apply the Zero Product Property, and then solve the equation.**

What are the roots of an equation? **the values of the variable that make the equation true**

EXPLAIN 1

Finding Real Solutions of Simple Quadratic Equations

INTEGRATE TECHNOLOGY

When zeros occur at non-integer x-values, using a table to find them can be difficult. Show students that they can use a graphing calculator to find zeros by graphing the function and selecting **2:zero** from the **CALCULATE** menu.

PROFESSIONAL DEVELOPMENT

Math Background

Students encounter many functions in their study of mathematics. The simplest are the parent functions, including the parent linear function $f(x) = x$, the parent quadratic function $f(x) = x^2$, the parent cubic function $f(x) = x^2$, and the parent square root function $f(x) = \sqrt{x}$. Each parent function is the building block for the functions in its family. Understanding the parent functions is a first step in understanding equations and graphs of functions in general. For advanced functions, there may be no true parent function. There is no one parent exponential function, although $f(x) = b^x$ may be considered the parent for any base b, $b > 0$, and $b \neq 1$.

AVOID COMMON ERRORS

When solving an equation of the form $x^2 = s$ where $s > 0$, students may look only for positive solutions. Be sure that students also look for negative solutions.

QUESTIONING STRATEGIES

What is rationalizing the denominator? **eliminating the radical from a denominator by multiplying by a form of 1**

EXPLAIN 2

Solving a Real-World Problem Using a Simple Quadratic Equation

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 A variable followed by a subscript zero, as in V_0 or h_0, usually indicates an initial value of the variable. The zero indicates the value of the variable when the time t is 0.

Example 1 **Solve the quadratic equation by taking square roots.**

(A) $2x^2 - 16 = 0$

Add 16 to both sides.	$2x^2 = 16$
Divide both sides by 2.	$x^2 = 8$
Use the definition of square root.	$x = \pm\sqrt{8}$
Use the product property.	$x = \pm\sqrt{4} \cdot \sqrt{2}$
Simplify.	$x = \pm 2\sqrt{2}$

(B) $-5x^2 + 9 = 0$

Subtract 9 from both sides.	$-5x^2 = \boxed{-9}$
Divide both sides by $\boxed{-5}$.	$x^2 = \boxed{\frac{9}{5}}$
Use the definition of square root.	$x = \pm\sqrt{\boxed{\frac{9}{5}}}$
Use the quotient property.	$x = \pm\boxed{\frac{\sqrt{9}}{\sqrt{5}}}$
Simplify the numerator.	$x = \pm\boxed{\frac{3}{\sqrt{5}}}$
Rationalize the denominator.	$x = \pm\boxed{\frac{3\sqrt{5}}{5}}$

Your Turn

Solve the quadratic equation by taking square roots.

3. $x^2 - 24 = 0$

$x^2 = 24$

$x = \pm\sqrt{24}$

$x = \pm 2\sqrt{6}$

4. $-4x^2 + 13 = 0$

$-4x^2 = -13$

$x^2 = \frac{13}{4}$

$x = \pm\frac{\sqrt{13}}{\sqrt{4}}$

$x = \pm\frac{\sqrt{13}}{2}$

Explain 2 Solving a Real-World Problem Using a Simple Quadratic Equation

Two commonly used quadratic models for falling objects near Earth's surface are the following:

- Distance fallen (in feet) at time t (in seconds): $d(t) = 16t^2$
- Height (in feet) at time t (in seconds): $h(t) = h_0 - 16t^2$ where h_0 is the object's initial height (in feet)

For both models, time is measured from the instant that the object begins to fall. A negative value of t would represent a time before the object began falling, so negative values of t are excluded from the domains of these functions. This means that for any equation of the form $d(t) = c$ or $h(t) = c$ where c is a constant, a negative solution should be rejected.

COLLABORATIVE LEARNING

Peer-to-Peer Activity

Have pairs of students work together to make note cards for each method they have learned to find the zeros of quadratic functions. Suggest that they describe the steps of the method as well as any advantages or disadvantages that they observed. Students can continue to add notes to the cards as they learn additional methods.

Example 2 **Write and solve an equation to answer the question. Give the exact answer and, if it's irrational, a decimal approximation (to the nearest tenth of a second).**

(A) If you drop a water balloon, how long does it take to fall 4 feet?

Using the model $d(t) = 16t^2$, solve the equation $d(t) = 4$.

Write the equation. $16t^2 = 4$

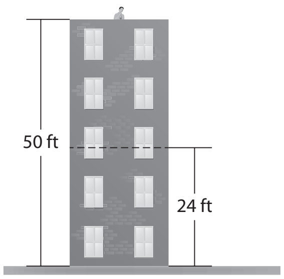
Divide both sides by 16. $t^2 = \frac{1}{4}$

Use the definition of square root. $t = \pm\sqrt{\frac{1}{4}}$

Use the quotient property. $t = \pm\frac{1}{2}$

Reject the negative value of t. The water balloon falls 4 feet in $\frac{1}{2}$ second.

(B) The rooftop of a 5-story building is 50 feet above the ground. How long does it take the water balloon dropped from the rooftop to pass by a third-story window at 24 feet?



Using the model $h(t) = h_0 - 16t^2$, solve the equation $h(t) = 24$. (When you reach the step at which you divide both sides by −16, leave 16 in the denominator rather than simplifying the fraction because you'll get a rational denominator when you later use the quotient property.)

Write the equation. $\boxed{50} - 16t^2 = \boxed{24}$

Subtract 50 from both sides. $-16t^2 = \boxed{-26}$

Divide both sides by −16. $t^2 = \boxed{\frac{26}{16}}$

Use the definition of square root. $t = \pm\sqrt{\boxed{\frac{26}{16}}}$

Use the quotient property to simplify. $t = \pm\boxed{\frac{\sqrt{26}}{4}}$

Reject the negative value of t. The water balloon passes by the third-story window in $\boxed{\frac{\sqrt{26}}{4}} \approx \boxed{1.3}$ seconds.

Reflect

5. **Discussion** Explain how the model $h(t) = h_0 - 16t^2$ is built from the model $d(t) = 16t^2$.

Think of $h(t)$, a falling object's height h at time t, as the distance that the object has *left* to fall. Since the total distance to fall is h_0, $h(t)$ is the distance left after subtracting the distance already fallen, $d(t)$, from h_0.

AVOID COMMON ERRORS

Because the word *maximum* is often associated with positive amounts, students may incorrectly assume that a quadratic function with a positive leading coefficient should have a maximum. Stress that, in fact, the "positive" parabola has a minimum and the "negative" parabola is the one with the maximum.

DIFFERENTIATE INSTRUCTION

Manipulatives

Students may benefit from using algebra tiles to practice factoring quadratic expressions. For example, show students how to model $x^2 + 6x + 8$ with algebra tiles. Then demonstrate that arranging the tiles in a rectangle models the product $(x + 2)(x + 4)$.

QUESTIONING STRATEGIES

What is the zero of a function? How does this translate when you use quadratic functions to model the height of a soccer ball after it is kicked? **The zero of a function is a value of the input x that makes the output $f(x)$ equal zero. In the case of a soccer ball, the zero of the function is the time it takes for the soccer ball to hit the ground after it is kicked.**

CONNECT VOCABULARY EL

Relate *imaginary unit* to real number or whole number units. Ask students to state what the square root of positive 1 is. Likewise, if the square root of –1 is the imaginary unit *i*, then its square is the original negative number (–1 in this case). Explain why imaginary units and imaginary numbers were invented—to define the square root of negative numbers.

EXPLAIN 3

Defining Imaginary Numbers

AVOID COMMON ERRORS

Some students may try to simplify a complex number by combining the real part and the imaginary part. For example, they may try to write $5 + 6i$ as $11i$. Emphasize that just as unlike terms in an algebraic expression cannot be combined, the real and imaginary parts of a complex number cannot be combined. Therefore, $5i + 6i = 11i$, but $5 + 6i \neq 5i + 6i$.

Your Turn

Write and solve an equation to answer the question. Give the exact answer and, if it's irrational, a decimal approximation (to the nearest tenth of a second).

6. How long does it take the water balloon described in Part B to hit the ground?

Using the model $h(t) = h_0 - 16t^2$, solve the equation $h(t) = 0$.

$$50 - 16t^2 = 0$$
$$-16t^2 = -50$$
$$t^2 = \frac{50}{16}$$
$$t = \pm\sqrt{\frac{50}{16}} = \frac{5\sqrt{2}}{4}$$

Reject the negative value of t. The water balloon hits the ground in $\frac{5\sqrt{2}}{4} \approx 1.8$ seconds.

7. On the moon, the distance d (in feet) that an object falls in time t (in seconds) is modeled by the function $d(t) = \frac{8}{3}t^2$. Suppose an astronaut on the moon drops a tool. How long does it take the tool to fall 4 feet?

Using the model $d(t) = \frac{8}{3}t^2$, solve the equation $d(t) = 4$.

$$\frac{8}{3}t^2 = 4$$
$$t^2 = \frac{3}{2}$$
$$t = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{6}}{2}$$

Reject the negative value of t. The tool falls 4 feet in $\frac{\sqrt{6}}{2} \approx 1.2$ seconds.

Explain 3 Defining Imaginary Numbers

You know that the quadratic equation $x^2 = 1$ has two real solutions, the equation $x^2 = 0$ has one real solution, and the equation $x^2 = -1$ has no real solutions. By creating a new type of number called *imaginary numbers*, mathematicians allowed for solutions of equations like $x^2 = -1$.

Imaginary numbers are the square roots of negative numbers. These numbers can all be written in the form bi where b is a nonzero real number and i, called the **imaginary unit**, represents $\sqrt{-1}$. Some examples of imaginary numbers are the following:

- $2i$
- $-5i$
- $-\frac{i}{3}$ or $-\frac{1}{3}i$
- $i\sqrt{2}$ (Write the i in front of the radical symbol for clarity.)
- $\frac{i\sqrt{3}}{2}$ or $\frac{\sqrt{3}}{2}i$

Given that $i = \sqrt{-1}$, you can conclude that $i^2 = -1$. This means that the square of any imaginary number is a negative real number. When squaring an imaginary number, use the power of a product property of exponents: $(ab)^m = a^m \cdot b^m$.

LANGUAGE SUPPORT EL

Connect Vocabulary

Provide students with 4 to 6 "square root cards"—index cards displaying the square roots of several negative and positive integers. Working in pairs, one student shows a card and the other student decides the kind of number it represents. If both agree, the first student then finds the *square* of that square root. Students switch roles and repeat the process until all the cards have been used.

Example 3 Find the square of the imaginary number.

Ⓐ $5i$

$$(5i)^2 = 5^2 \cdot i^2 = 25(-1) = -25$$

Ⓑ $-i\sqrt{2}$

$$(-i\sqrt{2})^2 = \left(\boxed{-\sqrt{2}}\right)^2 \cdot i^2 = \boxed{2}(-1) = \boxed{-2}$$

Reflect

8. By definition, i is a square root of -1. Does -1 have another square root? Explain.
Yes, $-i$ is also a square root of -1 because squaring $-i$ also gives -1.

Your Turn

Find the square of the imaginary number.

9. $-2i$

$$(-2i)^2 = (-2)^2 \cdot i^2 = 4(-1) = -4$$

10. $\frac{\sqrt{3}}{3}i$

$$\left(\frac{\sqrt{3}}{3}i\right)^2 = \left(\frac{\sqrt{3}}{3}\right)^2 \cdot i^2 = \frac{3}{9}(-1) = -\frac{1}{3}$$

Explain 4 Finding Imaginary Solutions of Simple Quadratic Equations

Using imaginary numbers, you can solve simple quadratic equations that do not have real solutions.

Example 4 Solve the quadratic equation by taking square roots. Allow for imaginary solutions.

Ⓐ $x^2 + 12 = 0$

Write the equation.	$x^2 + 12 = 0$
Subtract 12 from both sides.	$x^2 = -12$
Use the definition of square root.	$x = \pm\sqrt{-12}$
Use the product property.	$x = \pm\sqrt{(4)(-1)(3)} = \pm 2i\sqrt{3}$

QUESTIONING STRATEGIES

What is an imaginary number? **An imaginary unit i is defined as $\sqrt{-1}$. You can use the imaginary unit to write the square root of any negative number. Imaginary numbers can be written in the form bi, where b is a nonzero real number and i is the imaginary unit. $\sqrt{-1}$**

EXPLAIN 4

Finding Imaginary Solutions of Simple Quadratic Equations

QUESTIONING STRATEGIES

What do you look for during the solving process to indicate that a quadratic equation might have imaginary solutions? **The value x^2 is equal to a negative number or x is equal to the positive or negative square root of a negative number.**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

MP.3 Ask how the imaginary solutions of a quadratic equation of the form $ax^2 = c$ are related, and what their sum might be. Students should see that the solutions are opposites and should surmise that their sum is 0.

ELABORATE

CONNECT VOCABULARY EL

A *family of functions* is a set of functions whose graphs have basic characteristics in common. Functions that are in the same family are transformations of their parent function.

QUESTIONING STRATEGIES

How do you multiply powers with the same base when the exponents are rational? **Use the Product of Powers property or Quotient of Powers property and simplify.**

Explain why recognizing parent functions is useful for graphing. **Recognizing the parent function can help you predict what the graph will look like and help you fill in the missing parts.**

SUMMARIZE THE LESSON

How can you solve a quadratic equation by taking square roots? **Rewrite the equation so that the squared variable is isolated on one side of the equation, then take square roots of both sides. Use the fact that every positive real number has both a positive and a negative square root, and simplify the radical expression, using the product and quotient properties of square roots as necessary.**

(B) $4x^2 + 11 = 6$

Write the equation.	$4x^2 + 11 = 6$
Subtract 11 from both sides.	$\boxed{4}\, x^2 = \boxed{-5}$
Divide both sides by $\boxed{4}$.	$x^2 = \boxed{-\frac{5}{4}}$
Use the definition of square root.	$x = \pm\sqrt{\boxed{-\frac{5}{4}}}$
Use the qoutient property.	$x = \pm \boxed{\frac{\sqrt{5}}{2}i}$

Your Turn

Solve the quadratic equation by taking square roots. Allow for imaginary solutions.

11. $\frac{1}{4}x^2 + 9 = 0$

$\frac{1}{4}x^2 = -9$

$x^2 = -36$

$x = \pm\sqrt{-36}$

$x = \pm 6i$

12. $-5x^2 + 3 = 10$

$-5x^2 = 7$

$x^2 = -\frac{7}{5}$

$x = \pm\sqrt{-\frac{7}{5}}$

$x = \pm\frac{\sqrt{35}}{5}i$

Elaborate

13. The quadratic equations $4x^2 + 32 = 0$ and $4x^2 - 32 = 0$ differ only by the sign of the constant term. Without actually solving the equations, what can you say about the relationship between their solutions?
The first equation has imaginary solutions, while the second has real solutions, but the solutions will only differ by the factor of the imaginary unit *i*.

14. What kind of a number is the square of an imaginary number?
It is a negative real number.

15. Why do you reject negative values of t when solving equations based on the models for a falling object near Earth's surface, $d(t) = 16t^2$ for distance fallen and $h(t) = h_0 - 16t^2$ for height during a fall?
A negative value of *t* represents time before the fall, but both models deal only with time after the instant that the fall begins.

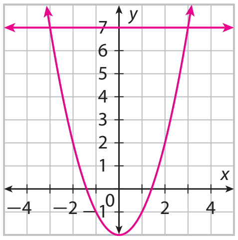
16. **Essential Question Check-In** Describe how to find the square roots of a negative number.
The square roots of a negative number, $-a$, are given by the imaginary unit *i* times the square roots of the corresponding positive number, *a*.

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

1. Solve the equation $x^2 - 2 = 7$ using the indicated method.

a. Solve by graphing.



Treat each side of the equation as a function, and graph the two functions, which in this case are $f(x) = x^2 - 2$ and $g(x) = 7$, on the same coordinate plane. The x-coordinates of the two points where the graphs intersect are $x = -3$ and $x = 3$.

b. Solve by factoring.

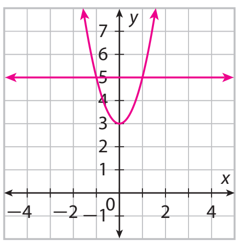
$$x^2 - 9 = 0$$
$$(x+3)(x-3) = 0$$
$$x + 3 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = -3 \quad \text{or} \quad x = 3$$

c. Solve by taking square roots.

$$x^2 = 9$$
$$x = \pm\sqrt{9}$$
$$x = \pm 3$$

2. Solve the equation $2x^2 + 3 = 5$ using the indicated method.

a. Solve by graphing.



Treat each side of the equation as a function, and graph the two functions, which in this case are $f(x) = 2x^2 + 3$ and $g(x) = 5$, on the same coordinate plane. The x-coordinates of the two points where the graphs intersect are $x = -1$ and $x = 1$.

b. Solve by factoring.

$$2x^2 - 2 = 0$$
$$2(x+1)(x-1) = 0$$
$$x + 1 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = -1 \quad \text{or} \quad x = 1$$

c. Solve by taking square roots.

$$2x^2 = 2$$
$$x^2 = 1$$
$$x = \pm\sqrt{1}$$
$$x = \pm 1$$

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
1–6	**1** Recall of Information	**MP.4** Modeling
7–10	**2** Skills/Concepts	**MP.4** Modeling
11–16	**2** Skills/Concepts	**MP.2** Reasoning
17–20	**2** Skills/Concepts	**MP.4** Modeling
21–22	**2** Skills/Concepts	**MP.5** Using Tools

EVALUATE

ASSIGNMENT GUIDE

Concepts and Skills	Practice
Explore Investigating Ways of Solving Simple Quadratic Equations	Exercises 1–2
Example 1 Finding Real Solutions of Simple Quadratic Equations	Exercises 2–6
Example 2 Solving a Real-World Problem Using a Simple Quadratic Equation	Exercises 7–10
Example 3 Defining Imaginary Numbers	Exercises 11–13
Example 4 Finding Imaginary Solutions of Simple Quadratic Equations	Exercises 15–18

AVOID COMMON ERRORS

When students draw the graph of a model of a quadratic function $F(x)$, they may draw a smooth curve through the points they plotted. Remind them that a reasonable domain of $F(x)$ consists of whole-number (or decimal) values. Therefore, the graph of the function is actually a set of discrete points rather than a smooth curve.

Solve the quadratic equation by taking square roots.

3. $4x^2 = 24$

$x^2 = 6$

$x = \pm\sqrt{6}$

4. $-\frac{x^2}{5} + 15 = 0$

$-\frac{x^2}{5} = -15$

$x^2 = 75$

$x = \pm\sqrt{75}$

$x = \pm 5\sqrt{3}$

5. $2(5 - 5x^2) = 5$

$10 - 10x^2 = 5$

$-10x^2 = -5$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}}$

$x = \pm\frac{\sqrt{2}}{2}$

6. $3x^2 - 8 = 12$

$3x^2 = 20$

$x^2 = \frac{20}{3}$

$x = \pm\sqrt{\frac{20}{3}}$

$x = \pm\frac{\sqrt{20}}{\sqrt{3}} = \pm\frac{\sqrt{60}}{3} = \pm\frac{2\sqrt{15}}{3}$

Write and solve an equation to answer the question. Give the exact answer and, if it's irrational, a decimal approximation (to the nearest tenth of a second).

7. A squirrel in a tree drops an acorn. How long does it take the acorn to fall 20 feet?

Using the model $d(t) = 16t^2$, solve the equation $d(t) = 20$.

$16t^2 = 20$

$t^2 = \frac{5}{4}$

$t = \pm\sqrt{\frac{5}{4}}$

$t = \pm\frac{\sqrt{5}}{2}$

Reject the negative value of t. The acorn falls 20 feet in $\frac{\sqrt{5}}{2} \approx 1.1$ seconds.

8. A person washing the windows of an office building drops a squeegee from a height of 60 feet. How long does it take the squeegee to pass by another window washer working at a height of 20 feet?

Using the model $h(t) = h_0 - 16t^2$, solve the equation $h(t) = 20$.

$60 - 16t^2 = 20$

$-16t^2 = -40$

$t^2 = \frac{10}{4}$

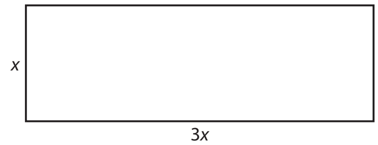
$t = \pm\sqrt{\frac{10}{4}}$

$t = \pm\frac{\sqrt{10}}{2}$

Reject the negative value of t. The squeegee passes by the other window washer in $\frac{\sqrt{10}}{2} \approx 1.6$ seconds.

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
23	1 Recall of Information	MP.2 Reasoning
24–25	3 Strategic Thinking H.O.T.	MP.4 Modeling
26	3 Strategic Thinking H.O.T.	MP.3 Logic

Geometry Determine the lengths of the sides of the rectangle using the given area. Give answers both exactly and approximately (to the nearest tenth).



9. The area of the rectangle is 45 cm².

$$(\text{length})(\text{width}) = \text{area}$$
$$(3x)(x) = 45$$
$$3x^2 = 45$$
$$x^2 = 15$$
$$x = \pm\sqrt{15}$$

Reject the negative value of x because length cannot be negative. So, the width of the rectangle is $\sqrt{15} \approx 3.9$ cm, and the length is $3\sqrt{15} \approx 11.6$ cm.

10. The area of the rectangle is 54 cm².

$$(\text{length})(\text{width}) = \text{area}$$
$$(3x)(x) = 54$$
$$3x^2 = 54$$
$$x^2 = 18$$
$$x = \pm\sqrt{18}$$
$$x = \pm 3\sqrt{2}$$

Reject the negative value of x because length cannot be negative. So, the width of the rectangle is $3\sqrt{2} \approx 4.2$ cm, and the length is $9\sqrt{2} \approx 12.7$ cm.

Find the square of the imaginary number.

11. $3i$

$$(3i)^2 = 3^2 \cdot i^2$$
$$= 9(-1)$$
$$= -9$$

12. $i\sqrt{5}$

$$(i\sqrt{5})^2 = (\sqrt{5})^2 \cdot i^2$$
$$= 5(-1)$$
$$= -5$$

13. $-i\frac{\sqrt{2}}{2}$

$$\left(-\frac{i\sqrt{2}}{2}\right)^2 = \left(-\frac{\sqrt{2}}{2}\right)^2 \cdot i^2$$
$$= \frac{1}{2}(-1)$$
$$= -\frac{1}{2}$$

Determine whether the quadratic equation has real solutions or imaginary solutions by solving the equation.

14. $15x^2 - 10 = 0$

$$15x^2 = 10$$
$$x^2 = \frac{2}{3}$$
$$x = \pm\sqrt{\frac{2}{3}}$$
$$x = \pm\frac{\sqrt{6}}{3}$$

The solutions are real.

15. $\frac{1}{2}x^2 + 12 = 4$

$$\frac{1}{2}x^2 = -8$$
$$x^2 = -16$$
$$x = \pm\sqrt{-16}$$
$$x = \pm 4i$$

The solutions are imaginary.

16. $5(2x^2 - 3) = 4(x^2 - 10)$

$$10x^2 - 15 = 4x^2 - 40$$
$$6x^2 = -25$$
$$x^2 = -\frac{25}{6}$$
$$x = \pm\sqrt{-\frac{25}{6}}$$
$$x = \pm\frac{5\sqrt{6}}{6}i$$

The solutions are imaginary.

QUESTIONING STRATEGIES

What is meant by reasonable domain? **A reasonable domain consists of the values of the independent variables that make sense in the context of the real-world situation.**

What is the domain of a function? Why might it differ from the reasonable domain? **The domain of a function is all the values of the independent variable for which the function is defined. It may include values that represent physically impossible situations, such as a nearly infinite number of minutes.**

Solve the quadratic equation by taking square roots. Allow for imaginary solutions.

17. $x^2 = -81$

$x = \pm\sqrt{-81}$

$x = \pm 9i$

18. $x^2 + 64 = 0$

$x^2 = -64$

$x = \pm\sqrt{-64}$

$x = \pm 8i$

19. $5x^2 - 4 = -8$

$5x^2 = -4$

$x^2 = -\frac{4}{5}$

$x = \pm\sqrt{-\frac{4}{5}}$

$x = \pm\frac{2\sqrt{5}}{5}i$

20. $7x^2 + 10 = 0$

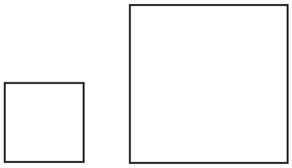
$7x^2 = -10$

$x^2 = -\frac{10}{7}$

$x = \pm\sqrt{-\frac{10}{7}}$

$x = \pm\frac{\sqrt{70}}{7}i$

Geometry Determine the length of the sides of each square using the given information. Give answers both exactly and approximately (to the nearest tenth).



21. The area of the larger square is 42 cm^2 more than the area of the smaller square.

Use the formula for the area A of a square with side length s: $A = s^2$.

$(2x)^2 = x^2 + 42$

$4x^2 = x^2 + 42$

$3x^2 = 42$

$x^2 = 14$

$x = \pm\sqrt{14}$

Reject the negative value of x. The smaller square has a side length of $\sqrt{14} \approx 3.7$ cm, and the larger square has a side length of $2\sqrt{14} \approx 7.5$ cm.

22. If the area of the larger square is decreased by 28 cm^2, the result is half of the area of the smaller square.

Use the formula for the area A of a square with side length s: $A = s^2$.

$(2x)^2 - 28 = \frac{1}{2}x^2$

$4x^2 - 28 = \frac{1}{2}x^2$

$\frac{7}{2}x^2 = 28$

$x^2 = 8$

$x = \pm\sqrt{8} = \pm 2\sqrt{2}$

Reject the negative value of x. The smaller square has a side length of $2\sqrt{2} \approx 2.8$ cm, and the larger square has a side length of $4\sqrt{2} \approx 5.7$ cm.

23. Determine whether each of the following numbers is real or imaginary.

a. i	☐ Real	X Imaginary
b. A square root of 5	X Real	☐ Imaginary
c. $(2i)^2$	X Real	☐ Imaginary
d. $(-5)^2$	X Real	☐ Imaginary
e. $\sqrt{-3}$	☐ Real	X Imaginary
f. $-\sqrt{10}$	X Real	☐ Imaginary

H.O.T. Focus on Higher Order Thinking

24. Critical Thinking When a batter hits a baseball, you can model the ball's height using a quadratic function that accounts for the ball's initial vertical velocity. However, once the ball reaches its maximum height, its vertical velocity is momentarily 0 feet per second, and you can use the model $h(t) = h_0 - 16t^2$ to find the ball's height h (in feet) at time t (in seconds) as it falls to the ground.

a. Suppose a fly ball reaches a maximum height of 67 feet and an outfielder catches the ball 3 feet above the ground. How long after the ball begins to descend does the outfielder catch the ball?

Using the model $h(t) = h_0 - 16t^2$, solve the equation $h(t) = 3$.

$$67 - 16t^2 = 3$$
$$-16t^2 = -64$$
$$t^2 = 4$$
$$t = \pm\sqrt{4}$$
$$t = \pm 2$$

Reject the negative value of t. The outfielder caught the ball 2 seconds after it reached its maximum height.

b. Can you determine (without writing or solving any equations) the total time the ball was in the air? Explain your reasoning and state any assumptions you make.

The other solution to the quadratic equation $h(t) = 3$, -2 seconds, is another time when the ball would have been 3 feet above the ground. This would have happened 2 seconds *before* the ball reached its maximum height. If you assume that the batter hit the ball at a height of 3 feet, then you can conclude that the ball was in the air for a total of 4 seconds.

PEER-TO-PEER DISCUSSION

Ask students to discuss with a partner a topic they would like to research to find a data set with time as the independent variable.

Students may find examples such as the length or weight of an animal as it grows; the populations of an endangered species in a city; the cost of a particular item; or a team's winning percentage. Have pairs of students then find information on the appropriate model for the data set.

JOURNAL

Have students write a journal entry that describes how they could apply their knowledge of graphs of quadratic functions to solve real-world problems.

25. **Represent Real-World Situations** The aspect ratio of an image on a screen is the ratio of image width to image height. An HDTV screen shows images with an aspect ratio of 16:9. If the area of an HDTV screen is 864 in^2, what are the dimensions of the screen?

The width of the screen must be some multiple of 16, and the height of screen must be the same multiple of 9. Let m be the common (positive) multiplier, so that the width is $16m$, the height is $9m$, and the ratio of width to height is $\frac{16m}{9m} = \frac{16}{9}$.

$$\begin{aligned} \text{(width)(height)} &= \text{area} \\ 16m \cdot 9m &= 864 \\ 144m^2 &= 864 \\ m^2 &= 6 \\ m &= \pm\sqrt{6} \end{aligned}$$

Reject the negative value of m. The width of the screen is $16\sqrt{6} \approx 39.2$ inches, and the height of the screen is $9\sqrt{6} \approx 22.0$ inches.

26. **Explain the Error** Russell wants to calculate the amount of time it takes for a load of dirt to fall from a crane's clamshell bucket at a height of 16 feet to the bottom of a hole that is 32 feet deep. He sets up the following equation and tries to solve it.

$$\begin{aligned} 16 - 16t^2 &= 32 \\ -16t^2 &= 16 \\ t^2 &= -1 \\ t &= \pm\sqrt{-1} \\ t &= \pm i \end{aligned}$$

Does Russell's answer make sense? If not, find and correct Russell's error.

No, the time should not be an imaginary number of seconds. His error was using a positive number to represent the "height" of the bottom of the hole. If the hole is 32 feet deep, the bottom is at −32 feet relative to ground level.

$$\begin{aligned} 16 - 16t^2 &= -32 \\ -16t^2 &= -48 \\ t^2 &= 3 \\ t &= \pm\sqrt{3} \end{aligned}$$

Reject the negative value of t. The dirt took $\sqrt{3} \approx 1.7$ seconds to reach the bottom of the hole.

Lesson Performance Task

A suspension bridge uses two thick cables, one on each side of the road, to hold up the road. The cables are suspended between two towers and have a parabolic shape. Smaller vertical cables connect the parabolic cables to the road. The table gives the lengths of the first few vertical cables starting with the shortest one.

Displacement from the Shortest Vertical Cable (m)	Height of Vertical Cable (m)
0	3
1	3.05
2	3.2
3	3.45

Find a quadratic function that describes the height (in meters) of a parabolic cable above the road as a function of the horizontal displacement (in meters) from the cable's lowest point. Use the function to predict the distance between the towers if the parabolic cable reaches a maximum height of 48 m above the road at each tower.

Use a coordinate plane where the x-axis is located at the level of the road, and the y-axis is located at the shortest vertical cable. In this coordinate system, the general form of the height function is $h(x) = ax^2 + k$. Since the height of the shortest vertical cable is 3 m, $k = 3$ and $h(x) = ax^2 + 3$.

To find the value of a, use the data for one of the vertical cables (other than the shortest one). For instance, substitute 1 for x and 3.05 for $h(x)$ and solve $h(x) = ax^2 + 3$ for a.

$$h(x) = ax^2 + 3$$
$$h(1) = a \cdot 1^2 + 3$$
$$3.05 = a + 3$$
$$0.05 = a$$

So, $h(x) = 0.05x^2 + 3$. Confirm that $h(2) = 3.2$ and $h(3) = 3.45$.

Set the height function equal to 48 and solve for x.

$$\begin{aligned} h(x) &= 48 \\ 0.05x^2 + 3 &= 48 \\ 0.05x^2 &= 45 \\ x^2 &= 900 \\ x &= \pm\sqrt{900} \\ x &= \pm 30 \end{aligned}$$

The two towers are at 30 m in opposite directions from the shortest vertical cable, so the distance between towers is $30 - (-30) = 60$ m.

QUESTIONING STRATEGIES

How can you tell without calculating whether a quadratic equation has imaginary roots? **Graph the equation, and if the graph does not intersect the x-axis, the solutions are imaginary.**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Technology

MP.5 A graphing calculator or spreadsheet can also be used to quickly evaluate expressions for many values of the variable. Use the table feature of a graphing calculator to evaluate an expression for different unknown values.

EXTENSION ACTIVITY

The supporting cable on a suspension bridge is in the shape of a parabola, but a cable suspended from both ends takes the shape of a *catenary*. Have students research online to compare the shapes of a parabola and catenary. Some students might be interested in the equation for a catenary, $y = \frac{a}{2}\left(e^{x/a} + e^{-x/a}\right) = a \cosh \left(\frac{x}{a}\right)$, where a is the vertical distance from the x-axis to the vertex. Another topic of interest related to suspension bridges is the Tacoma Narrows Bridge collapse. Have students do an Internet search to find footage of this dramatic event.

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.

0 points: Student does not demonstrate understanding of the problem.